

In-flight Calibration of the Cassini Accelerometer

Erin J. Maneri
Montana State University
Bozeman, Montana 59717

E. David Skulsky
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91109-8099

Allan Y. Lee*
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91109-8099

Introduction

The Cassini spacecraft was launched on October 15, 1997 by a Titan 4B launch vehicle. After an interplanetary cruise of almost seven years, it will arrive at Saturn in July 2004. Unlike both Voyager 1 and 2, which only flew by Saturn, Cassini will orbit the planet for at least four years. Major science objectives of the Cassini mission include investigations of: the configuration and dynamics of Saturn's magnetosphere, the structure and composition of the rings, the characterizations of several of Saturn's icy moons, and others¹.

The Interplanetary Cruise phase of the Cassini mission includes two flybys of Venus (April 1998 and June 1999) and flybys of Earth (August 1999) and Jupiter (December 2000). Most of the velocity change ($\Delta \vec{V}$) needed for the trip to Saturn is obtained from these planetary gravity assists, but Cassini must occasionally perform Trajectory Correction Maneuvers (TCMs) to "fine-tune" the trajectory during the Cruise and Tour phases of the mission.

Trajectory Correction Maneuvers may be performed using either a mono-propellant (hydrazine) system or a bi-propellant (nitrogen tetroxide and monomethyl hydrazine) system. The mono-propellant system (subsequently referred to as the Reaction Control System or RCS) consists of four Z-facing 1 N thrusters and four Y-facing 1 N thrusters and is used only for small TCMs (< 1 m/s) and for three-axis attitude control of the spacecraft. Large TCMs are performed using the bi-propellant system and one of its two 445 N gimbaled main engines².

RCS TCMs are terminated when the attitude control

flight software estimate of the achieved $\Delta \vec{V}$ magnitude (based on a software model of thruster performance) is equal to the commanded $\Delta \vec{V}$ magnitude. In contrast, main engine TCMs are terminated using ΔV measurements from a single-axis accelerometer. The accelerometer is not used to terminate RCS TCMs because the accelerometer is not sensitive enough to accurately measure ΔV imparted on the spacecraft by the RCS thrusters.

The Cassini accelerometer measures the total ΔV along a single axis which is closely aligned with the spacecraft Z axis. However, the main engine thrust vector is, in general, not aligned with spacecraft Z axis. Therefore, knowledge of the thrust vector direction in spacecraft body coordinates is used to "de-project" the ΔV measured along the accelerometer axis to $\Delta \vec{V}$ along the thrust. When the magnitude of the "de-projected" $\Delta \vec{V}$ is greater than or equal to the commanded ΔV , the TCM is terminated.

Goals

The primary goal of this study was to confirm the pre-flight estimate of the accelerometer's scale factor using an in-flight calibration algorithm. Although all of the TCMs to date have been highly accurate in burn duration, it is still important to check what influence the launch, temperature cycles, and other variables have on the scale factor. Also, this in-flight calibration algorithm would be valuable as a diagnostic tool for use in the event of a significant maneuver execution error (that can be traced to the accelerometer).

Problem Formulation

The $\Delta \vec{V}$ experienced by Cassini during a main engine TCM is measured via two independent means. On

* To whom all correspondence should be sent. M.S. 230-104, Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, CA 91109-8099, USA.

the ground, Doppler shift in the frequency of spacecraft tracking signals received by the ground receiving stations is used to estimate a component of $\Delta \vec{V}$ that is parallel to the Earth-to-spacecraft vector. Simultaneously, onboard the spacecraft, an accelerometer is used to estimate a component of $\Delta \vec{V}$ that is parallel to the spacecraft Z-axis. These measurements are related, and we can use the highly accurate Doppler data to estimate the scale factor of the accelerometer.

Let $\vec{a}(t) = [a_x(t), a_y(t), a_z(t)]^T$ denote the time-varying spacecraft acceleration vector, in m/s^2 , and given in the spacecraft body coordinate frame. For a short-duration TCM which consumes little fuel, the mass of the spacecraft remains nearly unchanged across the burn. As such, the acceleration components do not vary significantly with time. On the other hand, for a long-duration TCM (such as the 87.6 minute Deep Space Maneuver described below), the time variation of these acceleration components cannot be ignored in estimating the accelerometer scale factor. To first order, we assume that:

$$\begin{aligned} a_x(t) &= a_{xc} + m_x t \\ a_y(t) &= a_{yc} + m_y t \\ a_z(t) &= a_z(t) \end{aligned} \quad (1)$$

Here, a_{xc} and a_{yc} are the X and Y-axis components of the spacecraft acceleration, respectively, at the start time of the burn. Also, m_x and m_y denote the time rates of increase of the spacecraft acceleration components, along the X and Y axes, respectively. Furthermore, let us denote the unit vector of the Earth-to-spacecraft vector, in spacecraft body coordinates, by $\hat{r}(t) = [r_x(t), r_y(t), r_z(t)]^T$. The unit vector $\hat{r}(t)$ is computed using the estimated spacecraft trajectory and Earth ephemeris (both given in a J2000 inertial coordinate frame) and the on-board estimate of the spacecraft attitude relative to that inertial coordinate frame. In terms of the components of $\vec{a}(t)$ and $\hat{r}(t)$, the Doppler measurements, $V^D(t)$, in m/s , are given by:

$$\begin{aligned} \frac{d}{dt} \{V^D(t)\} &= \vec{a}(t) \cdot \hat{r}(t) = a_{xc} r_x(t) + m_x t r_x(t) \\ &\quad + a_{yc} r_y(t) + m_y t r_y(t) + a_z(t) r_z(t) \end{aligned} \quad (2)$$

The Z-axis component of the spacecraft acceleration vector, $a_z(t)$, is related to the accelerometer raw measurement, $V^{acc}(t)$ (in units of data number), by:

$$a_z(t) = K_{sf} \frac{d}{dt} \{V^{acc}(t)\} - K_{bias} \quad (3)$$

Here, K_{sf} (in m/s per data number) and K_{bias} (in m/s^2) denote the scale factor and bias of the accelerometer,

respectively. Before the start of each main engine TCM, the bias of the accelerometer is calibrated by observing the output of the accelerometer while the spacecraft is in a quiescent state.

If we eliminate $a_z(t)$ from the last two equations and integrate the resulting equation with respect to time from t_1 to t_2 , we obtain:

$$z = H \vec{p} \quad (4)$$

Here, $z = \{V^D(t_2) - V^D(t_1)\} + K_{bias} \int_{t_1}^{t_2} r_z(t) dt$, is the measurement, and $\vec{p} = [a_{xc}, a_{yc}, K_{sf}, m_x, m_y]^T$ is a 5×1 unknown parameter vector whose third component is the to-be-estimated accelerometer scale factor. The 1×5 matrix $H = [\int_{t_1}^{t_2} r_x(t) dt, \int_{t_1}^{t_2} r_y(t) dt, \int_{t_1}^{t_2} r_z(t) dt, \int_{t_1}^{t_2} r_x(t) t dt, \int_{t_1}^{t_2} r_y(t) t dt]$ is the observation matrix. The problem has now been expressed in a form to which numerous linear estimation techniques may be used to determine the unknown parameters³.

A Least-squares Estimation Methodology

Typically, we have some *a priori* knowledge, \vec{p}_{ap} , of the approximate magnitude of the components of \vec{p} even without the measurement data. Hence, we may exploit this knowledge to improve our estimate of \vec{p} . The magnitudes of the \vec{p}_{ap} components are determined as follows. Let $a(t_0)$ and $a(t_f)$ denote the accelerations of the spacecraft, in m/s^2 , 3.8 minutes after the start and 3.8 minutes before the termination of the 87.6 minute DSM, respectively. In this way, we do not use data given in both the ignition and termination transients of the maneuver. The mean time rate of increase of the spacecraft acceleration, in m/s^3 , is given by: $\bar{m} = [a(t_f) - a(t_0)] / [t_f - t_0]$. Let $\vec{w} = [w_x, w_y, w_z]^T$ denote the direction of the engine thrust vector at t_0 in the spacecraft body frame (which is available from telemetry). The *a priori* value of the accelerometer scale factor, \bar{K}_{sf} , is available from ground tests. Using these estimates of \bar{m} , $a(t_0)$, w_x , w_y , and \bar{K}_{sf} , we have: $\vec{p}_{ap} = [a(t_0) w_x, a(t_0) w_y, \bar{K}_{sf}, \bar{m} w_x, \bar{m} w_y]^T$.

A good estimate of \vec{p} , taking into account both the measurement \vec{z} , and the *a priori* knowledge \vec{p}_{ap} is the weighted-least-squares estimate of \vec{p} denoted by \vec{p}_{wls} . It is the parameter vector that minimizes the following cost functional³:

$$\begin{aligned} J &= \frac{1}{2} \{ (\vec{p} - \vec{p}_{ap})^T S^{-1} (\vec{p} - \vec{p}_{ap}) + \\ &\quad (\vec{z} - H \vec{p})^T R^{-1} (\vec{z} - H \vec{p}) \} \end{aligned} \quad (5)$$

In equation (5), the matrix S denotes the 5×5 diagonal error covariance matrix of \vec{p}_{ap} : $S =$

Table 1 Results of Robustness Tests

Robustness Tests	Nominal Value	Variation Range	K_{sf} min	K_{sf} max
K_{sf} Initial Guess [mm/s per dn]	2.02	1.82-2.22	1.9758	1.9763
Uncertainty in Initial Guess, S [%]	10	1-20	1.9644	1.9762
Number of Data Points (Fig. 2)	600	50-2400	1.0736	2.0646
Uncertainty in Data, R	σ_R^2	$0.01 \times \sigma_R^2 - 1.99 \times \sigma_R^2$	1.7339	1.9763

$E\{[\bar{p} - \bar{p}_{ap}][\bar{p} - \bar{p}_{ap}]^T\}$ where “E” is the Expected Value operator. Similarly, R denotes the $N \times N$ error covariance matrix of the measurement error \bar{e} ($= \bar{z} - H\bar{p}$): $R = E\{\bar{e}\bar{e}^T\}$. The “sizes” of S and R will adjust the estimate to within a certain region of the initial guesses and the data, respectively. Note that if \bar{p}_{ap} is a good estimate of \bar{p} , then S is small. Accordingly, a larger weighting matrix S^{-1} is used in the first quadratic term in J . Similarly, if the measurement \bar{z} is accurate, a larger weighting R^{-1} is used in the second quadratic term in J . The parameter vector \bar{p}_{wls} that minimizes J is obtained by setting $\partial J / \partial \bar{p}$ to zero. The result is³:

$$\bar{p}_{wls} = \bar{p}_{ap} + GH^T R^{-1} (\bar{z} - H\bar{p}_{ap}) \quad (6)$$

where the 5×5 matrix G is a symmetric, non-diagonal, positive-definite matrix, representing the covariance matrix of the error of the estimated parameter vector \bar{p}_{wls} . It is given by: $G^{-1} = S^{-1} + H^T R^{-1} H$.

Cassini Deep Space Maneuver

The Cassini Deep Space Maneuver (DSM), a 450 m/s main engine TCM, was successfully executed on December 2, 1998. The purpose of this maneuver was to lower perihelion so that Cassini would subsequently pass close to Venus for a gravity assist. This is the second largest TCM that Cassini will ever perform (Saturn Orbit Insertion, to be performed in July 2004, is the largest) and it represented a unique opportunity to test the algorithm described above.

As mentioned earlier, the accelerometer bias (K_{bias}) is estimated by monitoring the accelerometer output while the spacecraft is dynamically quiescent. The value of K_{bias} estimated immediately prior to the Deep Space Maneuver is 0.003067 m/s². Also, the first and last 3.8 minutes of the 87.6-minute DSM measurement data, involving either burn ignition or termination transients, are not used in estimating the accelerometer scale factor. Doppler data, ephemeris data, accelerometer data, etc., are not available at a constant rate. The least frequent of these rates is that of the accelerometer data which is available once every $\Delta T = 8$ seconds. To avoid interpolation errors, this sampling rate was used over the 80 minute data span. Hence, $N = 80 \times 60 / 8 = 600$.

For the DSM, *a priori* values of various parameters are estimated as follows. The engine thrust unit vector

at t_0 , \bar{w} , is $[-0.02565, -0.1324, -0.9909]^T$. The magnitude of the accelerometer scale factor determined via ground tests, \bar{K}_{sf} , is 0.00202066 m/s per data number. The spacecraft acceleration at t_0 , $a(t_0)$, is estimated to be 0.08 m/s² from telemetry. The estimated value of \bar{m} is 2.4682×10^{-6} m/s³. The estimation uncertainties of these *a priori* estimates are captured by the weighting matrix S . The diagonal elements of S , in ascending order and in appropriate units, are: $\{0.1 \times a(t_0) w_x\}^2$, $\{0.1 \times a(t_0) w_y\}^2$, 2.712×10^{-7} , $\{0.1 \times \bar{m} w_x\}^2$, $\{0.1 \times \bar{m} w_y\}^2$.

The measurement uncertainties associated with both the Doppler data and accelerometer data are captured by the matrix R . All the diagonal elements of R are identical and are given by $\sigma_R^2 = 2\sigma_{\text{doppler}}^2 + \sigma_{\text{bias}}^2 \times \bar{r}_z^2 \times \Delta T^2$. In our study, we use: $\sigma_{\text{doppler}} = 0.001125$ m/s, $\sigma_{\text{bias}} = 3.3 \times 10^{-5}$ m/s², the time-averaged value of $r_z(t) = \bar{r}_z = -0.5148$, and $\Delta T = 8$ seconds.

Results of Robustness Test and Discussion

As described earlier, an *a priori* estimate, \bar{p}_{ap} , with its estimation uncertainty S , and the measurement uncertainty of both the Doppler and accelerometer data, R , were used in producing a weighted-least-squares estimate of the accelerometer's scale factor. It is of interest to investigate the sensitivities of the resultant scale factor estimate with respect to \bar{p}_{ap} , S , and R . Another robustness test which investigates the dependency of \bar{p}_{wls} on the number of data points used in the estimation process was also performed.

The algorithm did prove to be rather robust with respect to both initial guesses and uncertainty of the initial guess (the covariance matrix is S), but less so towards uncertainty in the measurement data (the covariance matrix is R) and data frequency. With an initial guess that varies within the range of 1.82 to 2.22 mm/s per data number, Fig. 1 shows that the converged scale factor is 1.9760 mm/s per data number. Results obtained by varying the nominal 10% uncertainty in the initial guesses of components of \bar{p}_{ap} (e.g., the first component of S is $\{0.1 \times a(t_0) \times w_x\}^2$) are given in the second row of Table 1. Note that the converged scale factor value is not sensitive to variations in the initial guess uncertainty.

Figure 2 and the third row of Table 1 show that if only 600 equally spaced data are used, the converged

scale factor is 1.9760 mm/s per data number. However, if more data points are used (via linear interpolation of the accelerometer data), the converged scale factor actually approaches the ground-calibrated value of 2.020 mm/s per data number. On the other hand, if data are "thinned" artificially, the converged scale factor begins to deviate significantly from the 1.9760 mm/s per data number value. One conclusion from this robustness test is that a better estimate of the accelerometer scale factor is possible if the accelerometer is sampled more frequently than the current telemetry rate of once every 8 seconds. However, pre-launch, there was no plan to calibrate the accelerometer's scale factor, and an unnecessarily high telemetry rate for the acceleration measurements is a waste of resources.

All things considered, the algorithm yields a scale factor estimate that is about 2.2% below the ground-calibrated value of 2.020 mm/s per data number. Probable causes are: (1) spacecraft center-of-mass migration during the DSM burn wasn't modeled; (2) the small angular misalignment between the accelerometer axis and the spacecraft Z axis was ignored; and (3) the accelerometer is not mounted near the spacecraft's center-of-mass and will therefore detect a small centrifugal acceleration with any non-zero spacecraft angular rate during the DSM. Errors introduced by (1) and (2) could be minimized if knowledge of the center-of-mass migration and the accelerometer axis are incorporated in the formulated problem.

Conclusions

To the best of our knowledge, there aren't any published results on the in-flight scale factor calibration of an accelerometer used in past interplanetary missions. As such, this study fills a gap in the literature on the in-flight characterization of accelerometers. Considering the simplicity of the approach used in this study, the estimated value of the accelerometer's scale factor is surprisingly close to the ground-based value. Furthermore, one sensitivity test performed in this study suggests that an even better match is possible if the accelerometer's outputs are sampled more frequently. This factor should be emphasized in all future attempts to perform in-flight characterization of a spacecraft's attitude sensors. It is likely that we will apply the developed algorithm on data gathered from future main engine TCM burns. In this way, we can study the long-term stability of the accelerometer's scale factor in an actual space environment.

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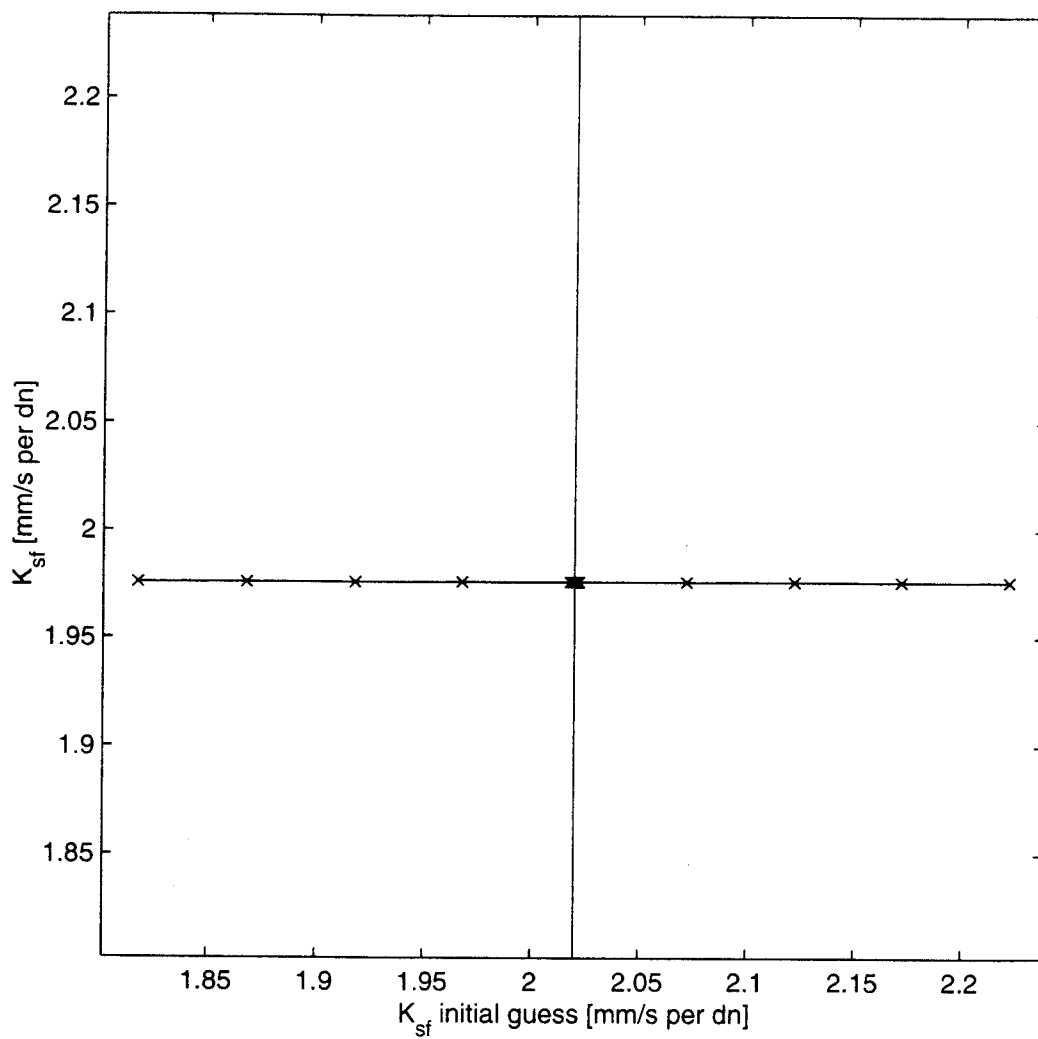


Fig. 1

Variation of the converged scale factor estimate with initial guesses

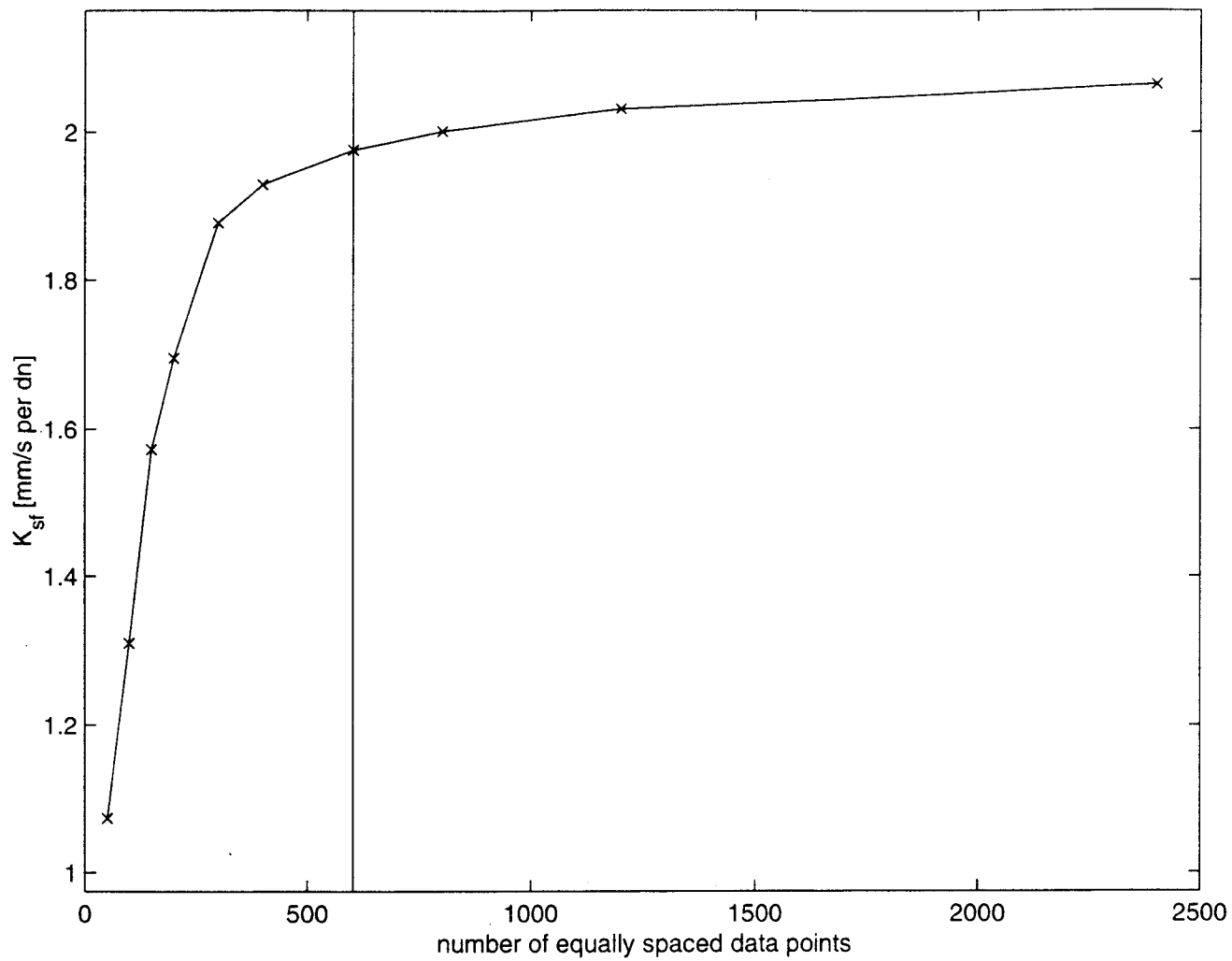


Fig. 2
Variation of the converged scale factor estimate with data size .